Sixth Annual Upper Peninsula High School Math Challenge

Northern Michigan University (Marquette Co, MI)

Saturday 14 March 2015

TEAM: SOLUTION

SCHOOL: _____

TEAM PROBLEMS **TIME: 45 minutes**

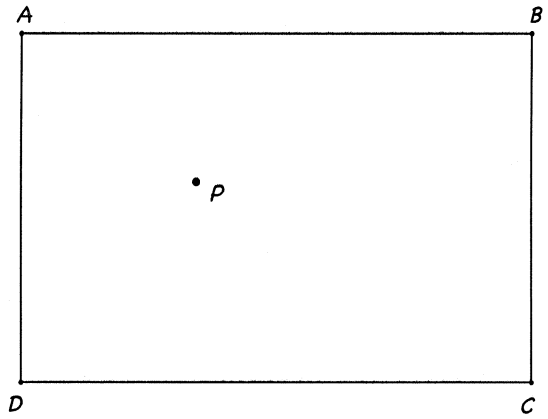
1. 6
2. 11
3. 64
4. 672
5. (3, -1)

Put no work on this side of the paper. Write the answers only in the above spaces. Put all work on the enclosed sheets of scrap paper, and hand in the scrap paper with your answer sheet.

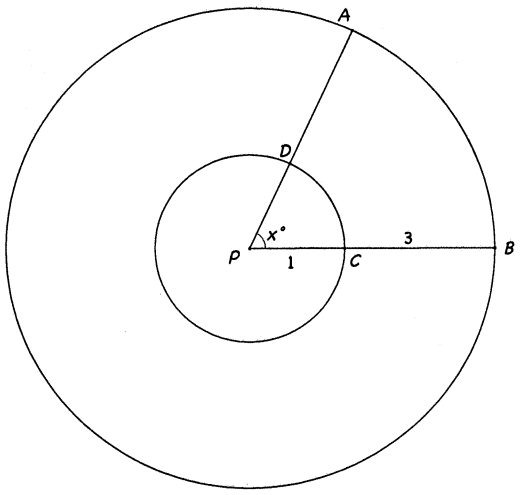
• 1. Find the value of x in simplified form:

$$x = \sqrt{17 - 12\sqrt{2}} + \sqrt{17 + 12\sqrt{2}}$$

• 2. P is a point in the interior of rectangle $ABCD$. The distances from P to three of the vertices are known: $PA = 5$ cm, $PB = 10$ cm, $PC = 14$ cm. In centimeters, what is the distance from P to vertex D?



• 3. Point P is the center of a pair of concentric circles and the area of region $ABCD$ (bounded by the segments and arcs in the diagram below) is $\frac{1}{6}$ of the area of the larger circle. Determine the measure, in degrees, of $\angle DPC$.



1. $X = \sqrt{17-12\sqrt{2}} + \sqrt{17+12\sqrt{2}}$

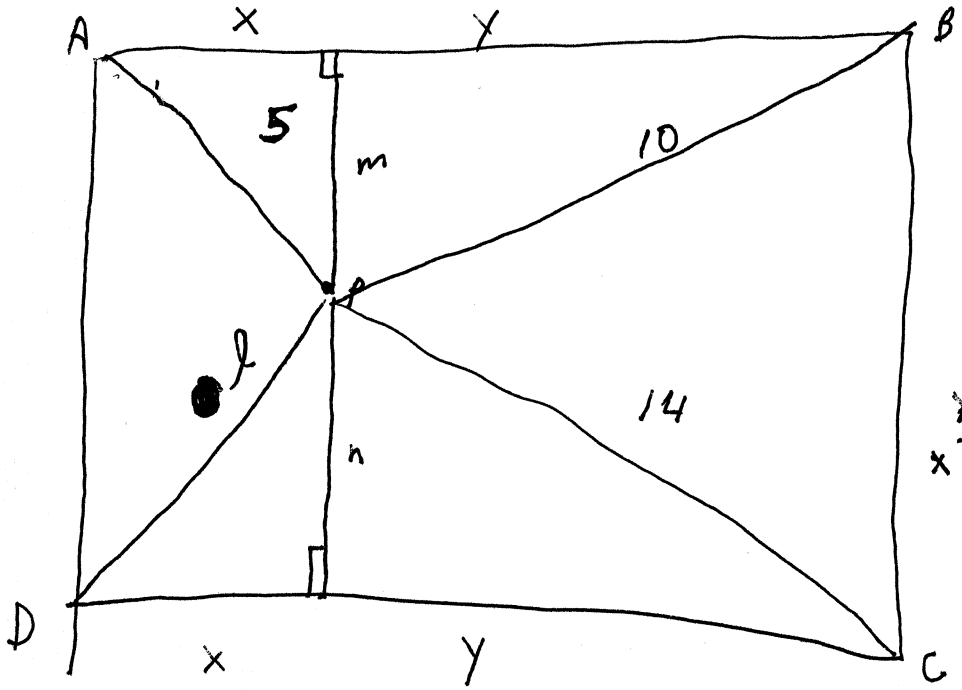
$X^2 = \left(\sqrt{17-12\sqrt{2}} + \sqrt{17+12\sqrt{2}}\right)^2$

$= 17-12\sqrt{2} + 2\sqrt{(17-12\sqrt{2})(17+12\sqrt{2})} + 17+12\sqrt{2}$

$= 34 + 2\sqrt{289-288} = 34+2\sqrt{1} = 36$

$X = 6$

2.



$y^2+m^2 = 10^2 = 100$

$x^2+m^2 = 5^2 = 25$

$y^2-x^2 = 75$

$y^2+n^2 = 14^2 = 196$

$x^2+n^2 = l^2$

$y^2-x^2 = 196-l^2$

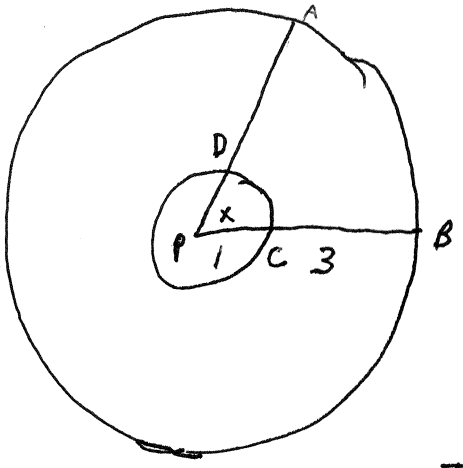
So

$196-l^2 = 75$

$l^2 = 196-75 = 121$

$l = 11$

3.



The area of the ~~sector~~ ρCD is

$$\pi(1^2)\cdot\frac{x}{360} = \frac{\pi x}{360}$$

The area of ρAB is

$$\pi(4^2)\cdot\frac{x}{360} = \frac{16\pi x}{360} = \frac{2\pi x}{45}$$

The area of $ABCD = \rho AB - \rho CD$ or

$$\frac{2\pi x}{45} - \frac{\pi x}{360} = \frac{15\pi x}{360} = \frac{\pi x}{24}$$

The larger circle has area $\pi 4^2 = 16\pi$

So, $\frac{\pi x}{24} = \frac{1}{6}\cdot 16\pi$

$\frac{\pi x}{24} = \frac{8}{3}\pi$

$\frac{x}{24} = \frac{8}{3}$

$\frac{3x}{24} = 8 \qquad \frac{x}{8} = 8 \qquad x = 64$

4. $1! = 1$

$2! = 2$

$3! = 2\cdot 3$

$4! = 2^3\cdot 3$

$5! = 2^3\cdot 3\cdot 5$

$6! = 2^4\cdot 3^2\cdot 5$

$7! = 2^4\cdot 3^2\cdot 5\cdot 7$

$8! = 2^7\cdot 3^2\cdot 5\cdot 7$

$9! = 2^7\cdot 3^4\cdot 5\cdot 7$

$\Pi = 2^{30}\cdot 3^{13}\cdot 5^5\cdot 7^3$

A perfect square can be divisible by

$2^0, 2^2, 2^4, \ldots 2^{30}$: 16 possible powers of 2

$3^0, 3^2, 3^4 \ldots 3^{12}$: 7 possible powers of 3

$5^0, 5^2, 5^4$: 3 possible powers of 5

$7^0, 7^2$: 2 possible powers of 7

$16\cdot 7\cdot 3\cdot 2 = 672$

5. $\frac{A}{x+2} + \frac{B}{2x-3} = \frac{A(2x-3)}{(x+2)(2x-3)} + \frac{B(x+2)}{(x+2)(2x-3)} = \frac{(2A+B)x + 2B-3A}{4x^2-3x-6} = \frac{5x-11}{4x^2-3x-6}$

$2A+B=5$

$2(3)+B=5$

$6+B=5$

$B=-1$

$2B-3A=-11$

$3A-2B=11$

$4A+2B=10$

$7A = 21$

$A=3$

$(3,-1)$